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EXPERIMENTAL STUDY AND CALCULATION OF HEAT AND MASS TRANSFER IN MOIST BODIES

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Results of an experimental study of heat and mass transfer involved in convective drying of a model material (quartz sand-bentonite mixture) are reported. The main characteristcics of the material are determined, namely, the curve of size distribution of capillaries, the curve of equilibrium states, etc. A method and experimental setups for determination of the thermophysical characteristics of the material and their dependences on moisture and temperature are discussed. A nonlinear system of differential equations of heat and mass transfer is solved by a numerical method. The numerical solution is compared to the experimentally obtained dependences $u(x, \tau)$ and $t(x, \tau)$.

Heat and mass transfer in colloid capillary-porous bodies is a complicated insufficiently investigated process that finds wide application in modern technological lines of material processing. Therefore investigation of such processes is of theoretical and practical significance. Several approaches exist for such investigations. In the approach developed by A. V. Luikov [1] heat and mass transfer involved in drying of colloid capillary-porous bodies is described by a system of differential equations, which for a one-dimensional problem and an infinite plate is as follows

$$R_c \frac{\partial T}{\partial \mathrm{Fo}^*} = \frac{\partial}{\partial x}\left(R_\lambda \frac{\partial T}{\partial x}\right) - \mathrm{Ko}^* R_r R_\varepsilon \frac{\partial U}{\partial \mathrm{Fo}^*},$$
$$\frac{\partial U}{\partial \mathrm{Fo}^*} = \mathrm{Lu}^* \frac{\partial}{\partial x}\left(R_{a_m} \frac{\partial U}{\partial x}\right) - \mathrm{Lu}^* \mathrm{Pn}^* \frac{\partial}{\partial x}\left(R_\delta \frac{\partial T}{\partial x}\right) \tag{1}$$

with the uniqueness conditions

$$T(x, 0) = 0, \quad U(x, 0) = 0, \tag{1a}$$

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = 0, \quad \left(\frac{\partial U}{\partial x}\right)_{x=0} = 0, \tag{1b}$$

$$-\left(R_\lambda \frac{\partial T}{\partial x}\right)_{x=1} + \mathrm{Ki}_q(\mathrm{Fo}) - \mathrm{Lu}^* \mathrm{Ko}^*/R_r(1 - R_\varepsilon)|_{x=1} \mathrm{Ki}_m(\mathrm{Fo}) = 0,$$
$$-\left(R_{a_m} \frac{\partial U}{\partial x}\right)_{x=1} + \mathrm{Pn}^*\left(R_\delta \frac{\partial T}{\partial x}\right)_{x=1} + \mathrm{Ki}_m(\mathrm{Fo}) = 0. \tag{1c}$$

Investigations of heat and moisture transfer involved in convective drying of moist bodies conducted at the Boris Kidrič-Vinča Institute for a chosen model material have permitted determination of the dependence of transfer coefficients on temperature and moisture. Moreover, system of equations (1) has been solved by analytical

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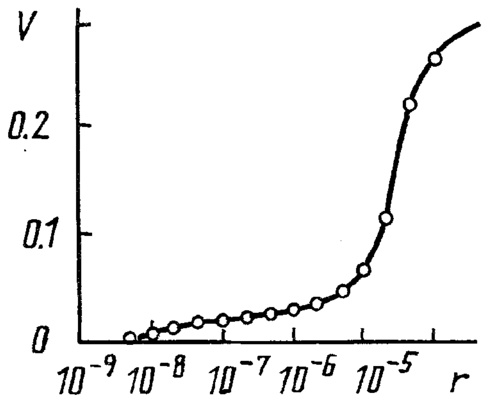


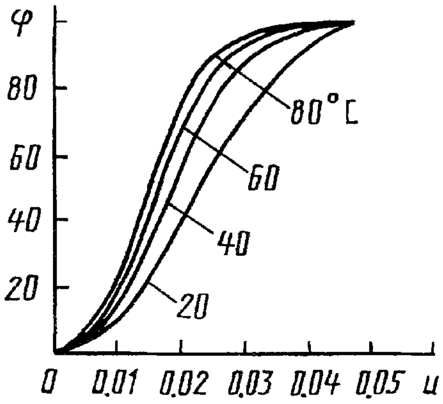


Fig. 1. Graph of radius distribution of capillaries. V, m^3/m^3; r, m.

Fig. 2. Sorption isotherms for a sand-bentonite mixture, $m_s/m_b = 4$. φ, %; u, kg/kg.

and numerical methods and the solutions obtained have been compared with experimental results. We report the main results of these studies.

Experiments were performed with a sand-bentonite mixture, as a model material, with the component ratio $m_s/m_b = 4$. This mixture with the initial moisture content was reduced, by pressing in a special mold, to a specified geometric volume (the density of the dry substance was $\rho_0 = 1500$ kg/m^3). A curve of the radius distribution of capillaries for the dried body is shown in Fig. 1. The body porosity is 38.8%, and more than 93% of the macrocapillaries have an equivalent diameter larger than 0.1 μm. With a change in the moisture content, the body porosity changes as well since the bentonite volume obeys the law $V/V_0 = 1 + 1.185u_b$, while the total volume of the body remains almost unchanged.

The specific surface of the material ($S_b = 10.7$ m^2/g) was determined by a modified BET-method at "one point" [2]. Using this value, we determined that at a moisture content of 1% three molecular layers of water exist. Therefore at $u > 0.01$ kg/kg the binding energy of moisture and the material may be considered insignificant compared to the heat of a phase transition.

Desorption isotherms were determined using an experimental setup in which air passes through a hygrothermostat and a layer of the material [3]. Equilibrium moisture was determined from the given parameters (t and φ) of moist air in relation to mass variation. Figure 2 presents desorption isotherms for the model material.

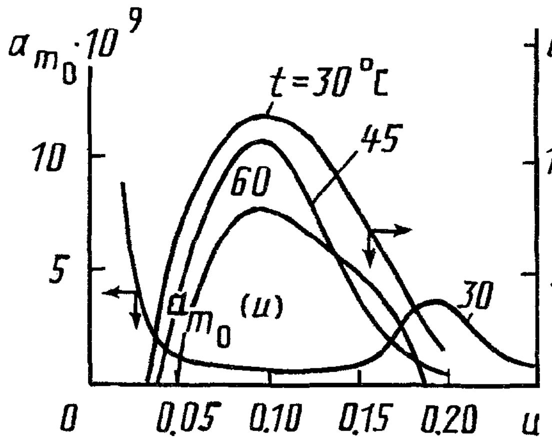
The specific heat of the dry substance of the model material was measured by a Bunsen calorimeter [4]. Here, direct measurements of the enthalpy of the sample were made as a function of the temperature. The specific heat of the moist material was determined by the formula

$$c = c_0 + c_u u = 612 + 1.93 \cdot 10^{-3} T^2 + 4178 \quad [\mathrm{J/(kg \cdot K)}].$$

To determine the thermal conductivity, we used the applied nonstationary method of a linear source [5], which is based on a heat conduction model as applied to a semi-infinite cylinder:

$$\lambda = (-862u^3 + 183u^2 + 0.54u + 1.2) \times (1 - 1.98 \cdot 10^{-3} t) \quad [\mathrm{W/(m \cdot K)}].$$

We used this setup with a cylindrical sample and a linear heat source to determine the thermal diffusivity. Three thermocouples were located along the radius. The differential equation of heat conduction $\partial T/\partial \tau = a\Delta T$ was solved numerically by the method of finite differences with boundary conditions that were determined by the first and third thermocouples. We took the value that agreed best with the measured and the calculated temperature with respect to position of the middle thermocouple. Owing to direct connection of the experimental setup to a computer the data obtained were qualitatively and rapidly processed. The experiment took 1–5 min, and for this time 4500 temperature values were recorded from four thermocouples (one thermocouple served for determination of λ and the remaining three – for calculation of a).



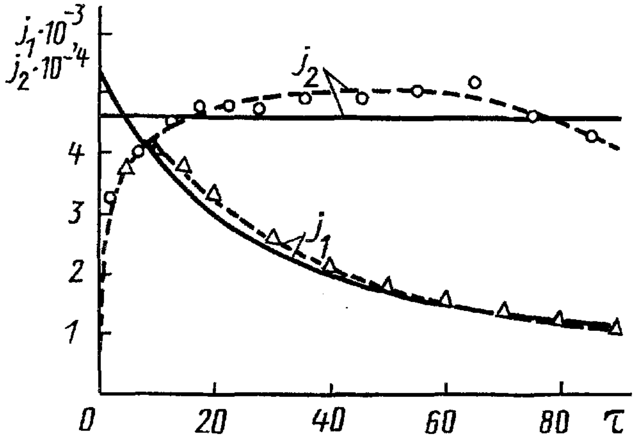


Fig. 3. Heat-gradient coefficient of diffusion versus moisture content. a_{m0}, m^2/sec; δ, kg/(kg·K).

Fig. 4. Flux density as a function of time. j_1, J/(m^2·sec); j_2, kg/(m^2·sec); τ, min.

The measured values of a are quite consistent with the calculated values: $a = \lambda/c\rho_0$, λ, c, and ρ_0.

The coefficient of moisture diffusion a_m was determined by the stationary method [6]. Figure 3 shows the dependences $a_{m0}(u)$ for $T_0 = 303$ K. The tempearture dependence of the diffusion coefficient may be represented by the equation $a_m/a_{m0} = (T/T_0)^n$, where $n = 10$.

The same figure shows the dependence of the heat-gradient coefficient determined by the stationary-flow technique [7].

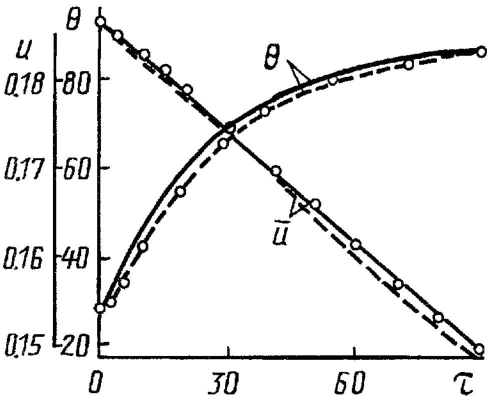
The fields of the temperature $t(x, \tau)$ and the moisture content $u(x, \tau)$, needed for the analysis of drying dynamics, and the densities of the moisture flow $j_p(\tau)$ and the heat flux $q_p(\tau)$ on a surface, needed for the kinetics analysis, were determined on the experimental setup [7]. The measuring cross section of the channel was rectangular ($H/h = 14$) with precisely determined profiles of temperatures and air velocities. Good insulation [8] of the side walls ensured process stability. Boundary conditions (1b) on the lower side of the sample were provided by insulation and a compensation heater. The change in the sample mass was traced on a balance. Seven thermocouples positioned along the sample axis permitted recording of the temperature fields. The data obtained were transferred onto a magnetic disc of a POP-11 computer. When the experiment was completed, the sample was withdrawn and cut into 10–15 discs, which were dried in a dryer. Then the moisture content profile was determined.

System of equations (1) with uniqueness conditions (1a), (1b), (1c) was solved analytically with the constant parameters $R_c = R_\lambda = R_\delta = R_{a_m} = 1$, $R_\varepsilon = \varepsilon = \text{const}$. The experimentally obtained dependences of the density of heat and moisture fluxes are described by the expressions

$$J_k = \sum_{i=1}^{1} a_{ki} \exp(-P_{ki}\tau), \qquad \mathrm{Ki}_k = \sum_{i=1}^{1} A_{ki} \exp(-P_{ki}\,\mathrm{Fo}), \quad k = 1, 2. \tag{2}$$

A program may be composed [8] to calculate $U(x, \mathrm{Fo})$, $T(x, \mathrm{Fo})$, $\overline{U}(\mathrm{Fo})$, $\overline{T}(\mathrm{Fo})$, $u(x, \tau)$, $t(x, \tau)$, $\overline{u}(\tau)$, $\overline{t}(\tau)$ if the values of λ, ρ, c, ε, δ, i, a_m and expression (2), i.e., a_{ki}, P_{ki}, are known.

System of nonlinear differential equations of heat and mass transfer (1) with uniqueness conditions (1a), (1b), (1c) was solved in the most general form by the numerical method of finite differences [7, 10]. For this, a special form based on the Crank–Nicolson method was used. Special subprograms were adopted to write the dependences of the thermophysical characteristics on u and t. Boundary conditions (1c) or the flow densities $j_i(\tau)$ may be prescribed experimentally or by any analytical expression. Figure 4 shows the measured flow densities for one experiment (E.3.4 [9]) and a dependence of type (2) for analytical representation of these values. In Figs. 5, 6, and 7 results of analytical and numerical solutions are compared with experimental values.



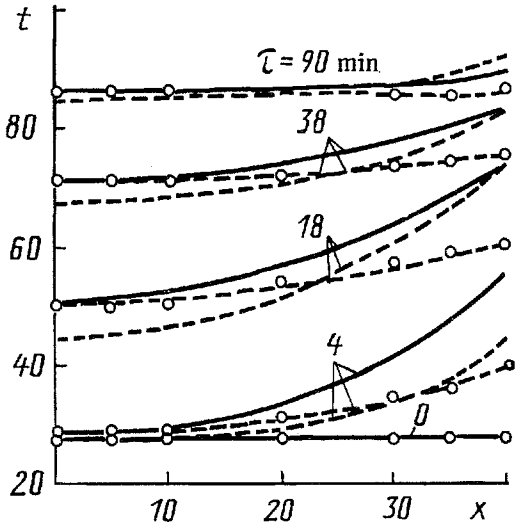


Fig. 5. Moisture content and temperature distributions (points, experimental; solid curves, analytical; dashed curves, calculated values). θ, °C.

Fig. 6. Temperature distribution with respect to the coordinate x for different moments of time, $\lambda = 2.1$ W/(m·K) (points, experimental; solid curves, analytical; dashed curves, calculated values). t, °C; x, mm

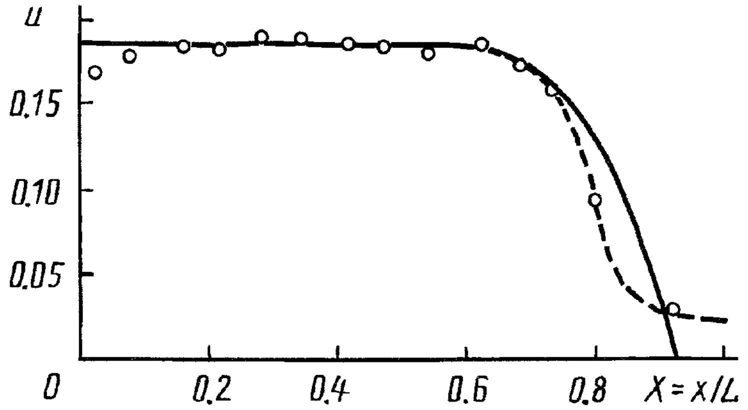


Fig. 7. Moisture field distribution after 90 min (points, experiment; solid curve, analytical, dashed curve, calculated).

As is seen in Fig. 6, the analytical and numerical solutions yield a temperature profile that differs from that measured by the mean temperature gradient. Although the agreement is rather good, it is necessary to analyze the reason for the differences observed. First of all, the method used to determine λ is based on a heat conduction equation that does not take into account the existence of a heat sink. Although the experiment lasts for a short time (1–5 min) a heat sink exists and the actual values of λ are higher than the measured ones. The second reason lies in the neglect of the term of convective heat transfer in system of equations (1) [1]. It is also seen (Fig. 6) that with these corrections for λ_{ef} the temperature profile coincides with the measured profile. The next reason for noncoincidence may be due to the fact that the boundary condition $(\partial T/\partial x) = 0$ has not been fulfilled exactly in the experiment. Recently, more detailed studies of heat transfer in moist materials and determinations of the coefficient λ have been carried out that may answer the questions arising in greater detail.

The profile of the moisture field determined by the analytical method (Fig. 7) differs from the measured one. The moisture value on a free surface is especially different. Coincidence of the numerical solution may be considered ideal (Fig. 7).

This completely confirms that the processes of heat and moisture transfer involved in convective drying of colloid capillary-porous bodies may be described accurately by nonlinear system of equations (1). In further studies a system of equations with boundary conditions of the third kind will be solved that opens wide opportunities for the analysis of transfer processes in the drying of moist bodies.

NOTATION

$\mathrm{Fo}^* = a_0\tau/L^2$, Fourier number; $\mathrm{Ki}_j = \mathrm{Ki}_q = j_1L/a_{m0}\rho_0u_0$, Kirpichev number for heat transfer; $\mathrm{Ki}_2 = \mathrm{Ki}_m = j_2L/a_{m0}\rho_0u_0$, Kirpichev number for mass transfer; $\mathrm{Ko}^* = r_0u_0/c_0t_0$, Kossovich number; $\mathrm{Lu}^* = a_{m0}/a_0$, Luikov number; L, sample length, m; $\mathrm{Pn}^* = \delta_0t_0/u_0$, Posnov number; $R_{a_m} = a_m/a_{m0}$, dimensionless diffusivity; $R_c = c/c_0$, dimensionless heat content; $R_v = v/v_0$, dimensionless specific heat of water evaporation; $R_\delta = a_m\delta/a_{m0}\delta_0$, dimensionless heat-gradient coefficient; $R_\lambda = \lambda/\lambda_0$, dimensionless thermal conductivity; $R_\varepsilon = \varepsilon$, criterion of phase change; T, temperature, K; $T^* = (t - t_0)/t_0$, dimensionless temperature; $U = (u_0 - u)/u_0$, dimensionless moisture content; $X = x/L$, dimensionless length; a, thermal diffusivity, $\mathrm{m}^2/\mathrm{sec}$; a_m, diffusion coefficient, $\mathrm{m}^2/\mathrm{sec}$; c, specific heat content, $\mathrm{J}/(\mathrm{kg}\cdot K)$; $j_1 = q_p$, specific heat flux over the surface, $\mathrm{W/m}^2$; $j_2 = j_p$, density of the moisture flow over the surface, $\mathrm{kg}/(\mathrm{m}^2\cdot\mathrm{sec})$; r, specific heat of water evaporation, J/kg; t, temperature, ^{0}C; $\bar{t}$, mean temperature of the body, ^{0}C; $\bar{u}$, mean moisture content of the body, kg/kg; u, moisture, kg/kg; x, coordinate, m; δ, relative coefficient of thermal diffusion, $\mathrm{kg}/(\mathrm{kg}\cdot\mathrm{K})$; λ, thermal conductivity, $\mathrm{W}/(\mathrm{m}\cdot\mathrm{K})$; ρ_0, density of the dry body, $\mathrm{kg/m}^3$; τ, time, sec.

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